Target aided online sensor localisation in bearing only clusters

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***Abstract*—In this work, we consider a network of bearing only sensors in a surveillance scenario. The processing of target measurements follow a two-tier architecture: The first tier is composed of centralised processing clusters whereas in the second tier, cluster heads perform decentralised processing. We are interested in the first tier problem of locating peripheral sensors relative to their cluster head. We mainly exploit target measurements received by the cluster head in a parameter estimation setting which involves Sequential Monte Carlo methods, and is known to have many difficulties in practice, including particle deficiency, sensitivity to initialisation, and high computational complexity. These difficulties are exacerbated by the bearing-only modality which provides a relatively poor target observability. We propose an online solution through Bayesian recursions on Junction Tree models of the posterior which partition the problem into fixed size subproblems and hence provides scalability with the number of sensors. We use the received signal strength as noisy range measurements to improve the robustness and accuracy of our algorithm. We demonstrate its efficacy with an example.**

I. Introduction

We consider networks of bearing only sensors for surveillance applications. Examples of such networks include passive sonar equipped surface or underwater buoys which communicate over a wireless network for underwater target detection, localisation and tracking [1], [2]. A single passive sensor is often not capable of delivering a reasonably good accuracy in target tracking which in turn necessitates the use of information from multiple geographically dispersed platforms. Multi-sensor exploitation in such networks can be facilitated by a hiearchical in-network processing structure the first tier of which is composed of local clusters, or, coalitions, performing centralised processing [3]. The cluster heads act as a fusion centre and process the target detections they collect from the peripheral nodes. A decentralised fusion paradigm (e.g., [4], [5]), then, can be used among cluster heads.

In this work, we are interested in locating the peripheral nodes in bearing only clusters. Sensor locations are crucial for processing the target measurements, because, these measurements are collected in the local coordinate systems of the sensors [2] and need to be mapped onto a common frame (the cluster head centered frame, in our case) before target positions can be estimated. Geographical routing algorithms underpinning the communication network also rely on a reasonably accurate knowledge of these locations [6].

We are particularly interested in problem settings in which the use of a global positioning system (GPS) is not possible or preferrable. For example, terresterial GPS sytems do not work effectively in underwater environments, mainly due to signal propagation constraints [1]. It is also not preferable for networks of surface platforms to rely on a GPS as it can easily become disfunctional under, for example, jamming. Other localisation techniques include those using communication network statistics [7] and/or following protocols with cooperative vehicles [8] which often fail to provide sufficient accuracy, low probability of intercept operation or facilitate self-organisation.

The main source of information we use is the detections already communicated to the cluster heads. When these measurements are considered alone, sensor localisation can be posed as parameter estimation in (latent) state space models (see, e.g., [9, Sec.IV]). Both Maximum Likelihood (ML) and Bayesian approaches to this problem use a "predictive" parameter likelihood estimated by filtering the target measurements using, for example, Sequential Monte Carlo (SMC) methods, and over a time window, integrating quantifications of how well the measurements are explained by the predictions. Online solutions obtained from either of the paradigms, however, suffer from a number of disadvantages such as sensitivity to initialisation, excessive computational load and bias [10].

The parameter space in sensor localisation is practically bounded, so, a non-informative localisation prior can, in principle, yield an ML equivalent Bayesian solution. In addition, the problem is rather well-behaved when individual nodes are capable of providing reasonably accurate target estimates as in the case of range-bearing sensors or cluster heads in the second tier: When a pair of nodes is considered, the location likelihood becomes informative in a short time window. Bayesian recursions realised via SMC and short time windows, then, lead to an online algorithm which, even with a simplified global model and approximate likelihoods chosen to comply with the system constraints, performs reasonably well [11]. In order to obtain an informative likelihood in the bearing only case, typically longer time windows and measurement histories of more than two sensors are needed. With increased window length and parameter dimensionality, however, SMC realisations become prone to particle deficiency (see, for example, [10] for a discussion of the topic). The dimensionality is also related to the sample size and adds to the issue of scalability with the number of sensors [12].

We propose an online Bayesian scheme which encompasses two features for a scalable and robust operation. First, we approximate the parameter posterior with a triangular Markov Random Field (MRF) and partition the global problem into fixed size subproblems. MRF models have proved useful for fusion in sensor networks [5], [13], and, in this case, the bounded dimensionality of these problems allows us to fix the sample sizes used as the number of sensors increases. The solutions with common location variables are implicitly combined according to inference rules over the Junction tree (JT) associated with the triangular graph [14]. Second, together with target measurements, we use the received signal strength (RSS) at the single element antenna of the cluster head as inaccurate range measurements collected when the target detections are received[1]. The combination of the JT approximations with RSS measurements allow us to achieve scalable, robust and fairly accurate localisation for bearing only sensors.

We provide the problem statement and a review of the online Bayesian approach in Section II. We introduce our approach in Section III an demonsrate it with an example in Section IV. Finally, we conclude in Section V.

II. Problem Definition

We consider a sensor cluster consisted of a cluster head and S peripherals(Fig. 1(a)). The cluster head is numbered as 0 and can communicate with peripheral $i \in \{1, \cdots, S\}$ through a single hop channel, for example, in the Very High Frequency (VHF) or Ultra High Frequency (UHF) band. All patforms are equipped with bearing only sensors which collect measurements from targets in the surveillance region.

We are interested in estimating the peripheral locations in the cluster head centered coordinate system. Let us denote the location of the i^{th} sensor by θ_i which takes values from a closed set $\Theta_i \subset \mathbb{R}^2$. The cluster parameter is the aggregation of all peripheral locations $\theta = [\theta_1, \cdots, \theta_S]^T$ such that $\theta \in \Theta$ where $\Theta = \Theta_1 \times \cdots \times \Theta_S$.

The measurements available at the cluster head for the estimation task are predominantly target detections from the peripherals and received signal strength at the receiver front-end[2]. In the following subsections, we introduce the parameter likelihood functions from these measurements.

A. Target measurements and the parameter likelihood

Let us assume the cluster head coordinate system as our natural reference frame. The peripherals and the cluster head collect measurements from a target with state x_k which is typically comprised of the target location and velocity, i.e., $x_k = [x_{k,1}, x_{k,2}, \dot{x}_{k,1}, \dot{x}_{k,2}]^T$, and evolves to the next step according to

$$x_{k+1} \sim \pi(.|x_k).$$

Sensor $i \in \{0, 1, ..., S\}$ measures z_k^i according to its local likelihood

$$z_k^i \sim p(.|\left[x_k\right]_i)$$
$$\left[x_k\right]_i = x_k - \theta_i.$$

[1]RSS measurements between several sensor pairs are useful for locating sensors, however, they might fail to provide sufficient accuracy for target tracking, when used alone [7]. Collecting many such measurements is also not preferable as this might compromise low probability of intercept operation.

[2]We assume that angle-of-arrival measurements for the received signal are not available as the transmission bands such as VHF might prohibit the use of an array of antennae on the sensor platforms.

We denote the likelihood of sensor i with its state argument in the natural coordinate frame by $p(z_k^i|x_k;\theta_i)$. For example, bearing only sensors are often modeled by

$$p(z_k^i|x_k;\theta_i) = \mathcal{N}(z_k^i - \arctan(x_{k,1} - \theta_{i,1}, x_{k,2} - \theta_{i,2}); 0, \sigma_i^2) \quad (1)$$

where $\mathcal{N}(.;0,\sigma_i^2)$ is a zero mean Gaussian with a variance equals to the second order moment of the measurement error.

Let us denote the measurement history of sensor i up to time k by $Z_{1:k}^i$. The parameter likelihood $l\left(Z_{1:k}^0, ..., Z_{1:k}^S|\theta\right)$ based on the cluster history is given by [9, Sec.IV]

$$l\left(Z_{1:k}^0, ..., Z_{1:k}^S|\theta\right) = \prod_{t=0}^{k-1} p\left(z_{t+1}^0, ..., z_{t+1}^S|Z_{1:t}^0, ..., Z_{1:t}^S, \theta\right) \quad (2)$$

where the factorisation follows from the chain rule. The factors of the product in the right hand side (RHS) of the equation above can be treated as instantaneous likelihoods for independent observations of θ.

In addition, the current measurements and the recent history are conditionally independent given the current target state and sensor locations, i.e., $\mathbf{z}_{t+1}^i \perp\!\!\!\perp \mathbf{Z}_{1:t}^i|\mathbf{x}_{t+1}, \boldsymbol{\theta}$ holds where $\perp\!\!\!\perp$ denotes the conditional independence relation and the random variables are written with bold letters. Let us denote the set of measurement histories $\{Z_{1:k}^0, ..., Z_{1:k}^S\}$ by $Z_{1:k}^{0:S}$. The instantaneous likelihood for t is, then, given by

$$p\left(z_{t+1}^0, ..., z_{t+1}^S|Z_{1:t}^{0:S}, \theta\right) = \int p(z_{t+1}^0|x_{t+1}) \prod_{i=1}^{S} p(z_{t+1}^i|x_{t+1};\theta_i) \times p(x_{t+1}|Z_{1:t}^{0:S}, \theta) \mathrm{d}x_{t+1}, \quad (3)$$

where the first terms inside the integral are measurement likelihoods and the last term is a prediction distribution for the target process at time $t+1$, based on the observation histories of all the nodes in the cluster until t. This distribution is output by the prediction stage of Bayesian recursive filtering with the location vector selected as θ.

B. Received Signal Strength measurements

We consider packet transmission from the peripherals to the cluster head carrying T target detections. We also assume that these detections are collected and transmitted synchronously, for the sake of simplicity.

The signal strength of the received packet from peripheral i follow a log-normal law for the distance [7] which is given by

$$l(P^i|\theta_i) = \mathcal{N}(P^i; \tilde{P}(\|\theta_i\|), \sigma_{dB}^2) \quad (4)$$
$$\tilde{P}(d) = P_0 - 10\mu \log \frac{d}{d_0} \quad (5)$$

where σ_{dB} is the standard deviation of the received power, $\|\theta_i\|$ is the norm of θ_i, $\tilde{P}(d)$ is the ensemble averaged power at distance d parameterised with the path-loss exponent μ and the reference power P_0 received at distance d_0. We assume that the RSS measurements of sensors are mutually independent.

C. Bayesian online solution with target measurements

We first consider using only the target measurements in a Bayesian framework. Integration of RSS likelihoods into this solution is relatively straightforward.

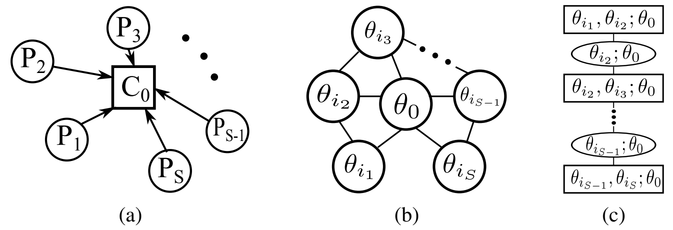


Fig. 1. (a) A bearing only cluster with S peripherals transmitting detections to their cluster head. (b) Triangular Markov Random Field $\mathcal{G}$ of location variables formed using an arbitary ordering $(i_1, i_2, ..., i_S)$ of peripherals. The cluster head is located at θ_0. (c) Junction tree $\mathcal{G}_T$ corresponding to the triangular MRF. The clique nodes are rectangles and separator nodes are ellipses.

Let us assume that $\boldsymbol{\theta}$ is a random vector with a prior distribution over Θ and density denoted by $p_0(\theta)$. The localisation posterior is, then, given by

$$p(\theta|Z^0_{1:k}, ..., Z^S_{1:k}) \propto p_0(\theta) l \left(Z^0_{1:k}, ..., Z^S_{1:k}|\theta\right) \tag{6}$$

where $l\left(Z^0_{1:k}, ..., Z^S_{1:k}|\theta\right)$ is the parameter likelihood. The sensor locations can be found by using the minimum mean squared error (MMSE) or the maximum a posteriori (MAP) estimation rules. The latter yields a solution equivalent to the Maximum Likelihood (ML) estimate when the prior is selected as the uniform distribution over Θ denoted by $U_\Theta(\theta)$.

A conceptual estimator based on Eq.s(3)–(6) implies batch processing of measurement histories. Online processing can be facilitated by time windowing measurements, introducing a slight dynamic evolution to $\boldsymbol{\theta}$ and using Bayesian recursions with this non-exact model to update a distribution over Θ:

$$p_n(\theta_n) \propto l(Z^{0:S}_{T(n-1)+1:Tn}|\theta_n) p_{n|n-1}(\theta_n) \tag{7}$$

$$p_{n|n-1}(\theta_n) = \int f_n(\theta_n|\theta_{n-1}) p_{n-1}(\theta_{n-1}) \mathrm{d}\theta_{n-1} \tag{8}$$

where $f_n(\theta_n|\theta_{n-1})$ models small Brownian motion steps, i.e.,

$$f_n(\theta_n|\theta_{n-1}) = \mathcal{N}(\theta_n - \theta_{n-1}; 0, \boldsymbol{I}\sigma^2_n) \tag{9}$$

where σ^2_n is the energy of the steps at n and $\boldsymbol{I}$ is the identity matrix with dimensionality equals to that of θ_n.

Such approximations have proved useful in self-localisation of range-bearing sensors [11] and have the potential to perform reasonably well for a wide range of parameter estimation problems in state-space models [10], including bearing only sensor localisation. We refer the reader to, e.g., [10], [15], and the references therein for other possible approaches and their criticism. In order to achieve a reasonable estimation accuracy using SMC Bayesian recursions, it is often necessary to use computationally involved techniques with relatively large particle sets (see, e.g., [16]) which makes it difficult to scale with increasing parameter dimensionality. In this work, we are interested in efficient and robust SMC computations for which we introduce a simplified probabilistic model for localisation which we discuss next.

III. Triangular MRF Localisation Posterior

In this section, we present our localisation posterior model, discuss inference on the model using Monte Carlo methods and prior shaping using RSS measurements.

We focus on the update equation (7) as it is the most computationally intensive step and it is relatively straightforward to design a numerical recipe for realising (8). Let us ignore the time subscripts for the clarity of discussion in this section and rewrite (7) as

$$p(\theta) \propto l(Z^{0:S}|\theta) p_0(\theta). \tag{10}$$

First, we assume that the individual peripheral locations are independent before the update, i.e.,

$$p_0(\theta) = \prod_{i \in \{1,2,...,S\}} p_{0,i}(\theta_i) \tag{11}$$

where subscript i indicates the fields of θ associated with peripheral i.

Second, we construct a triangular MRF $\mathcal{G}$ given $S \geqslant 2$ peripherals. A simple construction follows from an arbitrary ordering $(i_1, ..., i_S)$ of the peripherals and selecting 3-cliques as

$$\mathcal{C} \triangleq \{(\theta_0, \theta_{i_1}, \theta_{i_2}), (\theta_0, \theta_{i_2}, \theta_{i_3}), \cdots, (\theta_0, \theta_{i_{S-1}}, \theta_{i_S})\}. \tag{12}$$

Consequently, the size of all of the maximal cliques of $\mathcal{G}$ is three (Fig. 1(b)). Here, θ_0 has a density with all the probability mass at the origin.

We assert the assumption that the Markov properties of $\mathcal{G}$ hold for the localisation posterior given by Eq.(10). Conversely, $\mathcal{G}$ is an undirected graphical model for Eq.(10) which implies a particular factorisation for the posterior distribution. Specifically, for every triangular graph, there corresponds a Junction Tree (JT) $\mathcal{G}_T$ which is a tree structure composed of i) nodes associated with the (maximal) cliques of $\mathcal{G}$ (which are 3-cliques, for $\mathcal{G}$) connected through ii) separator nodes associated with variables common to both sides of the connection [14](Fig. 1(c)). The edges of a JT satisfy the running intersection property: Two nodes with common variables are connected through nodes which are associated with the same variables along the unique path connecting them. These properties lead to the distributions that satisfy the Markov properties of a JT factorise as [14]

$$p(\theta) = \frac{\prod_{c \in \mathcal{C}} p(\theta_c)}{\prod_{s \in \mathcal{S}} p(\theta_s)} \tag{13}$$

where $\mathcal{C}$ and $\mathcal{S}$ are the sets of clique nodes and separator nodes of the JT, respectively.

The JT we construct by selecting $\mathcal{C}$ as given in (12) forms a chain, in particular, with the set of separators given by

$$\mathcal{S} \triangleq \{(\theta_0, \theta_{i_k}) | k = 2, \cdots, S-1\}. \tag{14}$$

Consequently, our assumption implies factorisation of Eq.(10) as in (13). Together with the independence of the priors, the update in Eq.(10) becomes

$$p(\theta) \propto \frac{\prod_{k=1}^{S-1} l(Z^0, Z^{i_k}, Z^{i_{k+1}}|\theta_{i_k}, \theta_{i_{k+1}})}{\prod_{k=2}^{S-1} l(Z^0, Z^{i_k}|\theta_{i_k})} \prod_{i=1}^{S} p_{0,i}(\theta_i). \tag{15}$$

The factors in the numerator of (15) involve the joint paramater space for two peripherals and require three sensor histories (for the cluster head, peripherals i_k and i_{k+1}). The terms in the denominator involves only the cluster head and peripheral i_k. This structure provides the benefit of bounding the dimensionality of the SMC updates. Node marginals over this chain can be found by smoothing operations described by the JT algorithm which has a sum product structure [17].

A. Particle methods for inference on the JT localisation model

The JT algorithm over $\mathcal{G}_T$ can be realised using non-parametric belief propagation [18] which would result with samples generated from the posterior marginals of all cliques. These operations involve the costly step of taking the product of two distributions except for the variables not associated with any separator node. In order to avoid this, we sample only from $\boldsymbol{\theta}_{i_1}$ and $\boldsymbol{\theta}_{i_S}$ in an update step and use a different ordering of peripherals, for example, a one place cyclic permutation, in the next round of the Bayesian recursions. In other words, we update a different pair of location distributions at each step.

The first marginal of the JT posterior in (15) denoted by $p(\theta_{i_1})$ can easily be found as nested integrations of products which equivalently can be computed by a series of message passings after substituting $\mathcal{G}_T$ in the JT algorithm [14]. Specifically, the iterative scheme starting with

$$m_{S-1}(\theta_{i_{S-1}}) \triangleq \frac{1}{l(Z^{0,i_{S-1}}|\theta_{i_{S-1}})} \times \int_{\Theta_S} l(Z^{0,i_{S-1},i_S}|\theta_{i_{S-1}},\theta_{i_S}) p_{0,i_{S-1}}(\theta_{i_{S-1}}) p_{0,i_S}(\theta_{i_S}) \mathrm{d}\theta_{i_S} \tag{16}$$

and, continuing for $s = S-2, ..., 2$ with

$$m_s(\theta_{i_s}) \triangleq \frac{1}{l(Z^{0,i_s}|\theta_{i_s})} \times \int_{\Theta_{s+1}} l(Z^{0,i_s,i_{s+1}}|\theta_{i_s},\theta_{i_{s+1}}) p_{0,i_s}(\theta_{i_s}) m_{s+1}(\theta_{i_{s+1}}) \mathrm{d}\theta_{i_{s+1}} \tag{17}$$

leads to the marginal seeked after given by

$$p(\theta_{i_1}) \propto \int_{\Theta_{i_2}} l(Z^{0,i_1,i_2}|\theta_{i_1},\theta_{i_2}) p_{0,i_1}(\theta_{i_1}) m_2(\theta_{i_2}) \mathrm{d}\theta_{i_2}. \tag{18}$$

The same computational structure leads to $p(\theta_{i_S})$ when the messaging order is reversed to start at the first clique of $\mathcal{G}_T$.

Now, we describe Monte Carlo methods for computing (16)–(18). Suppose we are given particle sets $\{\omega_i^{(j)}, \theta_i^{(j)}\}_{j=1:M}$ representing $p_{0,i}(\theta_i)$ for all peripherals $1, ..., S$. An empirical distribution approximately proportional to (16) can be found using the Importance Sampling (IS) principle [19] as

$$\begin{aligned} m_{S-1}(\theta_{i_{S-1}}) &\underset{\sim}{\propto} \sum_{j=1}^{M} \tilde{\zeta}_{i_{S-1}}^{(j)} \delta(\theta_{i_{S-1}} - \theta_{i_{S-1}}^{(j)}) \\ \tilde{\zeta}_{i_{S-1}}^{(j)} &= \left(1/\sum_{j=1}^{M} \zeta_{i_{S-1}}^{(j)}\right) \zeta_{i_{S-1}}^{(j)} \\ \zeta_{i_{S-1}}^{(j)} &= \frac{\omega_{i_{S-1}}^{(j)}}{l(Z^{0,i_{S-1}}|\theta_{i_{S-1}}^{(j)}) \sum_{j'=k_1}^{k_N} \omega_{i_S}^{(j')}} \times \\ &\sum_{j'=k_1}^{k_N} \omega_{i_S}^{(j')} l(Z^{0,i_{S-1},i_S}|\theta_{i_{S-1}}^{(j)}, \theta_{i_S}^{(j')}) \end{aligned} \tag{19}$$

where $\{k_1, ..., k_N\}$ is an index set found by N times sampling from the discrete distribution implied by $\{\omega_{i_S}^{(j)}\}$.

Here, $l(Z^{0,i_{S-1}}|\theta_{i_{S-1}}^{(j)})$ is estimated using Eq.s(2)-(3) and an SMC filter on the measurement windows of the cluster head and peripheral i_{S-1}. The location of sensor i_{S-1} is taken as the particle value $\theta_{i_{S-1}}^{(j)}$. The other likelihood term $l(Z^{0,i_{S-1},i_S}|\theta_{i_{S-1}}^{(j)}, \theta_{i_S}^{(j')})$ is estimated similarly using, this time, the measurement windows of both peripherals i_{S-1} and i_S, and with that of the cluster head.

Note that the particle set $\{\tilde{\zeta}_{i_{S-1}}^{(j)}, \theta_{i_{S-1}}^{(j)}\}_{j=1:M}$ represents a probability density (approximately) proportional to m_{S-1}. Next, we use this sample set for approximating (17) for $s = S-2$. In general, given an IS approximation for m_{s+1}

$$m_{s+1}(\theta_{i_{s+1}}) \underset{\sim}{\propto} \sum_{j=1}^{M} \tilde{\zeta}_{i_{s+1}}^{(j)} \delta(\theta_{i_{s+1}} - \theta_{i_{s+1}}^{(j)}) \tag{20}$$

a similar approximation to $m_s(\theta_{i_s})$ can be found by, first, replacing $\{\omega_{i_S}^{(j)}, \theta_{i_S}^{(j)}\}$ and $\{\omega_{i_{S-1}}^{(j)}, \theta_{i_{S-1}}^{(j)}\}$ in (19) with $\{\omega_{i_{s+1}}^{(j)}, \theta_{i_{s+1}}^{(j)}\}$ and $\{\tilde{\zeta}_{i_s}^{(j)}, \theta_{i_s}^{(j)}\}$, respectively, and, second, using the measurement windows of the corresponding sensors to obtain the unnormalised weights $\zeta_s^{(j)}$s.

Finally, the marginal distribution (18) is updated using

$$\begin{aligned} p(\theta_{i_1}) &\approx \sum_{j=1}^{M} \omega_{i_1}^{(j)} \delta(\theta_{i_1} - \theta_{i_1}^{(j)}) \\ \omega_{i_1}^{(j)} &= \left(1/\sum_{j=1}^{M} \zeta_{i_1}^{(j)}\right) \zeta_{i_1}^{(j)} \\ \zeta_{i_1}^{(j)} &= \frac{\omega_{i_1}^{(j)}}{\sum_{j'=k_1}^{k_N} \tilde{\zeta}_{i_2}^{(j')}} \sum_{j'=k_1}^{k_N} \tilde{\zeta}_{i_2}^{(j')} l(Z^{0,i_1,i_2}|\theta_{i_1}^{(j)}, \theta_{i_2}^{(j')}) \end{aligned} \tag{21}$$

For each set of peripheral measurement windows received, we realise the update step of the Bayesian recursions given by (7) with the particle methods described above building upon the triangular localisation model and inference on the associated JT.

B. Prior shaping with RSS measurements

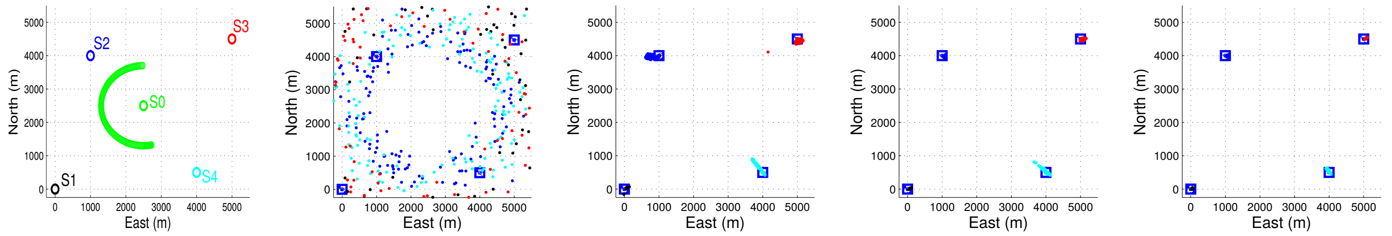
Informative location priors significantly help to improve the performance of Bayesian estimation in the type of problems considered in this work [10]. We use the RSS measurements for this purpose: At every measurement window step n, before we proceed with the update step (7) using the MC computations described in Section III-A, we find the marginal posteriors based on the RSS measurement using the likelihood given by (4).

Suppose we are given $\{\omega_i^{(j)}, \theta_i^{(j)}\}$ representing marginals of $p_{n|n-1}(\theta_n)$. Given the RSS measurement P_i, the IS weights of the location distribution are updated with

$$\begin{aligned} \omega_i^{(j)} &\leftarrow \left(1/\sum \tilde{\omega}_i^{(j)}\right) \tilde{\omega}_i^{(j)}, \\ \tilde{\omega}_i^{(j)} &= \omega_i^{(j)} l(P_i|\theta_i^{(j)}), \end{aligned} \tag{22}$$

where $\leftarrow$ denotes the assignment of the value at the RHS.

The RSS updates provide the additional benefit of conditioning the marginal distributions for the independence assumption used in the JT update since the RSS measurements are independent.



(a) (b) (c) (d) (e)

Fig. 2. (a) Illustration of the example scenario. (b) Scatter plot of the localisation prior at $n = 1$ after the first RSS update, prior to the JT update. (c) Localisation posteriors at $n = 5$, (d) $n = 10$, and (e) $n = 15$.

IV. Example

We demonstrate the proposed online scheme in an example scenario with five bearing only sensors collecting measurements from a target moving along a circular path with $20m/s$ constant speed (Fig. 2(a)). The noise standard deviation is $\sigma_i = 0.5$ for all sensors. The cluster head (sensor 0) receives new recent measurement windows of length $T = 10$ in every 10 time steps. The associated RSS measurements are modelled with Eq.s(4) and (5).

We use 150 particles for each peripheral and $N = 40$ cross terms in MC computations (Eq.s(19) and (21)). For each set of detections, we first sample from the prediction distribution given by (8) with the Markov shift in (9). Then, we update the predictions with the RSS measurements and use the particle sets representing location marginals in the JT update described in Section III-A. We initiate (i_1, i_2, i_3, i_4) with the natural ordering and use one place cyclic permutation at each step n. For example, at $n = 1$ sensor pairs $(1, 4)$, at $n = 2$ sensor pairs $(2, 1)$, and at $n = 3$ sensor pairs $(3, 2)$ are updated with target measurement windows collected between time steps 1–10,11–20, and 21–30, respectively.

In Fig. 2(b)–(e), we present the scatter plots of equally weighted particles representing localisation marginals at time steps $n = 0, 5, 10$ and 15 of a typical run. The mean distance error at the end of $n = 20$ steps is 45.7m which is %2.15 of the nearest peripheral distance (2121.3m of sensor 2).

V. Conclusion

In this work, we proposed an online scheme for finding the location of peripheral sensors in a bearing only cluster based primarily on target detections transmitted to the cluster head. Such estimation problems have many difficulties including particle deficiency, sensitivity to initialisation, and scalability. These problems are aggravated by the bearing only modality. Our main contribution is a Junction Tree model which addresses the scalability issue by enabling us to solve fixed size subproblems and combine the results in a rigorous framework. We circumvented the particle deficiency problem by using relatively small measurement windows in recursive updates. We exploit received signal strength measurements to provide informative priors to the JT update. We demonstrate the efficacy of our approach in an example. Future work includes a thorough comparison of the proposed scheme with other Bayesian techniques and investigation of simultaneous use of multiple Junction trees.

Acknowledgment

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) grants EP/J015180/1 and EP/K014277/1, and the MOD University Defence Research Collaboration in Signal Processing.

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